Physics 7B

DLM 17 Overview

Model: Simple Harmonic Motion

Act 7.2.1 The Universal Behavior of Oscillating Objects (~ 60 min)

Learning Goals:

- practice describing a variety of oscillatory motions in everyday words
- realizing what is common to the description of all oscillatory motions
- understanding how the position coordinate of an oscillating object varies as a function of time and what a graph of the position coordinate would look like
- understanding the precise meaning of the technical terms (amplitude, period, frequency) used to describe oscillatory motion

AC-7.2.2 Describing Simple Harmonic Motion Mathematically (~40 min)

Learning Goals:

- to make sense of the expression $y(t) = A \sin(2\pi t/T + \phi)$
- to understand how this expression describes the motion of a vibrating object
- to be able to get numerical values appearing in the expression from measurements made on a physical system.

Act 7.2.3 Describing Simple Harmonic Motion Mathematically II (~40 min)
(Expansion of parts (4) and (5) of Activity 7.2.2 and DLM 16 FNTs)

Learning Goals:

- to make sense of the expression $y(t) = A \sin(2\pi t/T + \phi)$
- to understand how this expression describes the motion of a vibrating object
- to really understand the meaning and function of the fixed phase constant, ϕ
- to be able to get numerical values appearing in the expression from measurements made on a physical system.

Announcements

- Keep studying for the **final**. Do a little bit each day by going over the FNTs, quizzes, DLMs, and practice problems (on the webpage with partial solutions).

- Review Sessions will be posted on the final exit handout.

- **Final exam will be given at 10:30 am on Monday March 19**. You must bring a picture ID. Show up at least 20 minutes earlier, so you can get a good seat and start on time and not wait in line.

The Universal Behavior of Oscillating Objects

(A) Do:

Set up and observe how each of the following three systems behaves when the object is set into motion.

Systems: 1) Pendulum: A mass (hook weight) hanging on a string.
2) Mass and Spring: A mass (hook weight) hanging vertically on a spring.
3) Metal meter stick: The meter stick is clamped to the table so 50 - 90 cm protrudes from the table edge.

Write: Each person must write out his or her own responses to these questions. You will need this for some FNTs. Respond to questions (a)-(d) below with a couple of sentences for each of the three systems without using physics language. Put these everyday language responses **on the board** and be prepared to share them with the whole class.

a) What you had to do to get the object to move.
b) What the resulting motion is like.
c) What aspect of the motion you could change depending on **how you started it** moving and what aspect(s) of the motion did not appear to depend on how you started it moving.
d) How the motion changed (or did not change) when you **physically changed the system** in the following ways (actually make the changes and observe them)
   1) Pendulum: change the length of the string, by at least a factor of two
      change the mass hanging on the string
   2) Mass and Spring: change the mass, by at least a factor of two
   3) Meter stick: change the length sticking out past the edge of the table
e) Discuss with your table partners what is common across the three systems about your responses to each of the four questions (a)-(d).

Whole Class Discussion

(B) Think, Discuss, and Write:

- Rewrite **on the board** the response you just wrote in (A) using the words ***amplitude***, ***period***, ***oscillation***, and ***force***, instead of some of the everyday words you used in (A).
- Write out on the board a definition of each of these four words.

Whole Class Discussion

(C) Think, Discuss, and Draw:

a) Sketch a graph **on the board** of the y-position coordinate for the mass spring system as a function of time when it is oscillating. Let $y = 0$ correspond to the equilibrium position of the mass. Show on your graph the amplitude, A, and period, T. **(Leave graphs on board for next activity)**
b) On the same axis, draw a second graph (use a dashed line) that corresponds to starting the oscillation with a different pull.
c) On the same axis, draw a third graph (using a different length dash) that corresponds to using the same pull as the first, but with a much smaller mass.
d) Describe in technical words the similarities and the differences in the three graphs you just made.

Whole Class Discussion

Describing Simple Harmonic Motion Mathematically

A) Starting the mass-spring at different places.

1) Look at the graphs you made in the previous activity. Let's let the y position of the mass on the spring be measured from its equilibrium position with the positive direction as up. **According to your graph**, where was the mass and what was it doing at the time t = 0.

2) Make two graphs (with the same amplitude and on the same time axis) of the motion that results when you i) release the mass from its lowest point at t = 0 and ii) release the mass from its lowest point but call t = 0 the time reaches the equilibrium point moving upward.

3) Discuss in your group how you would write a mathematical expression for each of the graphs you made in (2). Use general symbols (*A, T*), not numerical values. Hint: use a sine function for one and a cosine for the other. Be careful with minus signs. Think about what the "angle" of the sine or cosine function has to be. What happens when an angle goes through 2π radians? Remember, an angle has to be in units of degrees or radians. We want it in radians. **Write your expressions on the board.**

Whole Class Discussion

B) Turning "every equation" (that describes simple oscillation) into a sine function

4) a) Discuss in your group how you could write both expressions in (3) using only a sine function, by adding a constant angle to the angle involving the period and time. Call this constant angle ϕ (often called the **phase angle**, it depends on the initial conditions, e.g., the y-value at t = 0). Make sure everyone in your group understands how this works. Write a sine function with the actual value of ϕ next to each graph (reference p. 91 in your course notes).

   b) Two ways to graphically think of ϕ: (1) Which way (left or right) does the sine function "shift" if + or – ϕ is added? OR, (2) Which way does the y-axis "shift" if + or – ϕ is added? Check this with your expressions in 4a above.

5) Draw a graph and write an expression for the following situation: mass released from its lowest point but take t = 0 for when the mass is passing through equilibrium point ***moving down.***

Whole Class Discussion

Describing Simple harmonic Motion Mathematically II

A) Practice making it all make sense (FNTs 2 & 3)

1) Put your response to FNT 2 on the board. For each of the three cases, I, II, and III, make sure you have a clear statement responding to the sentence: "Explain in words how you knew how to "start" each graph where t = 0.".

2) Put your response to FNT 3 on the board. Make sure everyone in your group understands FNTs 2 and 3.

B) Connecting to real oscillations

3) Start each of the three oscillating systems in two different positions (your instructor might tell you specifically which positions to use). Take whatever measurements you need to make, so you can write out a mathematical expression using a sine function that describes the oscillation as a function of time with actual values instead of symbols. Indicate (perhaps with a sentence) which direction is positive as well as stating what the motion and position was when t = 0. Write these expressions on the board

4) Using your mass-spring system's data, write an expression (with actual numbers) that gives the position of the mass as a function of time using a phase constant $\phi = 2\pi/5$. Describe in words the motion of the mass at t = 0 (its approximate position and its direction of motion).

Whole class discussion

C) Connecting the parameters in the equations to motion (FNTs 4-6)

1) **FNT 4 & 5**. Discuss in your group how you found relationships of the parameters needed to understand these two questions. Only write responses on the board if you are unsure and want help from your instructor.

2) After discussing your response to **FNT 6** as a group, draw these three motion pictures of the mass-spring on the board in a vertical column. Label positions of amplitude, A, and draw a velocity vector on the mass to indicate its direction of motion. Write a descriptive oscillatory equation for each case.

Physics 7B **Exit Handout** **DLM 17**

FNTs

1) An expression for the velocity of a vibrating object can be obtained in the standard way by differentiating with respect to time the expression for the position of the object.

a) Differentiate with respect to time the general expression for $y(t)$ to get $v(t)$: $v(t) = dy/dt$ = ???

b) Plot (sketch) both $y(t)$ and $v(t)$ on the same time axes. (vertical scales will be different) Explain how ***values*** of $v(t)$ as read from your graph are related to features of the graph of $y(t)$.

c) Evaluate the expression you just got in (a) for the velocity for the particular case described in FNT 3 (FNT 2 case II for t = 0 and t = 1.25 s)

2) An expression for the acceleration of a vibrating object can be obtained in the standard way by differentiating with respect to time the expression for the velocity of the object (which is the same as taking the 2^{nd} derivative of the position of the object).

a) Differentiate w/ respect to time the expression for $v(t)$ from (4a) to get $a(t)$: $a(t) = dv/dt$ = ???

b) Plot (sketch) $y(t)$, $v(t)$ and $a(t)$ on the same time axes. (vertical scales will be different for each)

c) Explain, using your graphs, how $a(t)$ is related to $y(t)$. Be precise.

d) Mathematically, from (4a) and the general expression for $y(t)$, you can write $a(t)$ = -(constant) $y(t)$. What is the constant equal to?

3) This last expression we just found, $a(t)$ = -(constant) $y(t)$, is fundamental to simple harmonic motion. Any physical system for which this is true will oscillate in SHM and the displacement of the object that oscillates is given by the general expression for $y(t)$ at the top of the page. State in your own words the meaning of this relationship for the net force, ΣF, acting on an object that is oscillating in SHM. Hint: recall Newton's 2^{nd} law!

4) A pendulum of length r with a weight m is raised to an angle θ_{max} and when released will oscillate for a period T.

a. Write out an appropriate equation that describes its oscillation using the variables above.

b. Draw a large arc to show the path of the pendulum's mass. Label three representative times for the motion of the pendulum during 0 to T/2: i) the instant it is released (call this point A), ii) when it is at equilibrium (point B), and iii) somewhere in between equilibrium and the maximum amplitude on its way up (point C). Draw a second arc of the same size, label the exact same places, but now call them D, E, and F. The second sketch shows the motion from T/2 to T.

c. To the side of the sketch, draw a force diagram for each point. Are any diagrams the same?

d. When is the pendulum moving fastest? Slowest? How does this velocity relate to the position graph (Hint: refer to the graphs of position and velocity from previous FNTs).

e. On your sketch from part (b), draw appropriately scaled velocity vectors from points A, B, C, D, E, and F.

f. Explain how you might determine appropriate acceleration vectors for points A-F. Can you think of more than one way?

g. If the positions you chose, A-F, were each chosen to be t = 0, what could the phase constant ϕ be in each case?