Circuits

Physics 7B - Lecture 7

Prof. Conway

Bernoulli's Principle

- "The energy density in a fluid in equilibrium is the same everywhere."
- We express this in the form of Bernoulli's Equation:

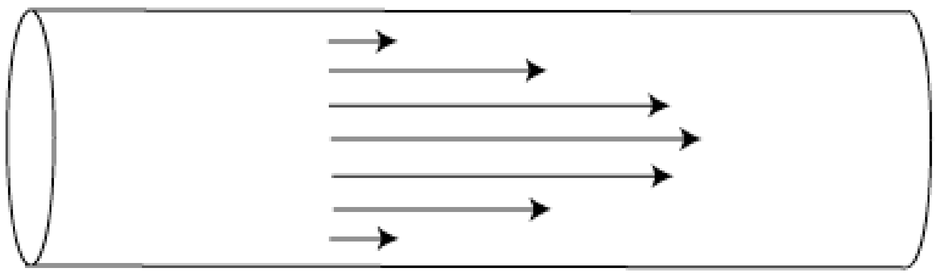
$$P + \rho g h + \frac{1}{2}\rho v^2 = constant$$

or, comparing two points in a fluid path:

$$\Delta P + \Delta(\rho g h) + \Delta(\frac{1}{2}\rho v^2) = 0$$

Fluid Resistance

- in real pipes, fluid flows more slowly next to walls:



- there is a resistance R leading to energy loss in pipe wall

- energy lost ΔE_{th} is proportional to current I:

$\Delta E_{th}/vol = IR$

Fluid Resistance - PRS

A 100-foot garden hose has resistance to water, such that it loses 10 J/s at a flow rate of 0.001 m^3/s. What is the resistance, including the units?

A. 0.01 J-m^3/s

B. 0.01 kJ-s/m^6

C. 10,000,000 J/s-m^6

D. 0.0001 m^3/J

E. 10^7 J-s/m^6

Units of R

- all terms in Bernoulli equation are in units of energy/volume
- I: volume/second ⇒ R: $J\text{-}s/m^6$
- resistance typically independent of I

Pumps

- placing a pump in a fluid circuit can change pressure (but not always!)
- flow rate into pump = flow rate out
- it takes a certain amount of energy per unit volume to do this: $\Delta E_{pump}/vol$
- need to have term in Bernoulli's equation
- when you analyze a pump, simply remember thaton the output side, the energy density goes up by $\Delta E_{pump}/vol$

Extended Bernoulli Eqn.

- adding the dissipative terms and pumps we have

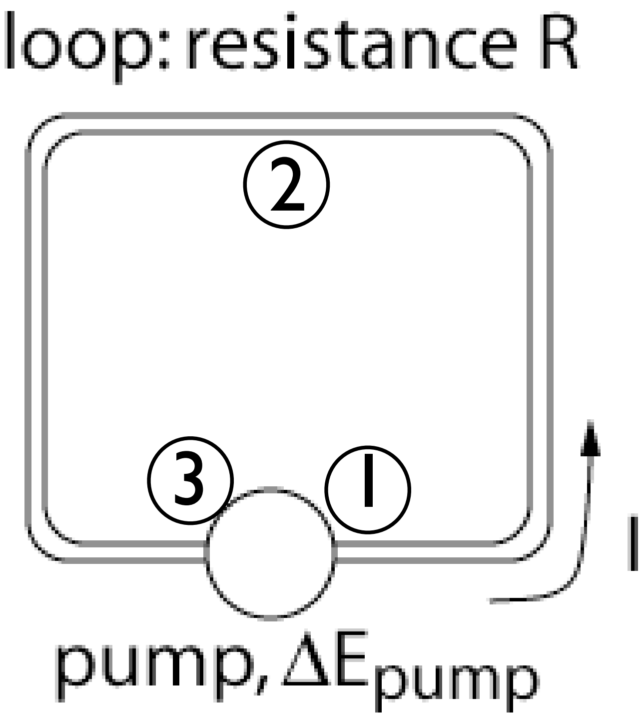
$$\Delta P + \Delta(\rho g h) + \Delta(\frac{1}{2}\rho v^2) = \frac{\Delta E_{pump}}{vol} - IR$$

- simple example: pump with circuit

Example: Simple Pump

- have a pump in a fluid circuit: what is the flow rate given R?

$$\Delta P + \Delta(\rho g h) + \Delta(\frac{1}{2}\rho v^2) = \frac{\Delta E_{pump}}{vol} - IR$$



$$\begin{aligned} P_2 - P_1 &= -IR/2 \\ P_3 - P_2 &= -IR/2 \\ P_1 - P_3 &= \Delta E_{pump}/vol \end{aligned}$$

$$\Delta E_{pump}/vol = IR$$

Power

- very specific definition in physics!

power = energy/time

- unit of power: Watt (1 W = 1 J/s)
- fluid circuit:

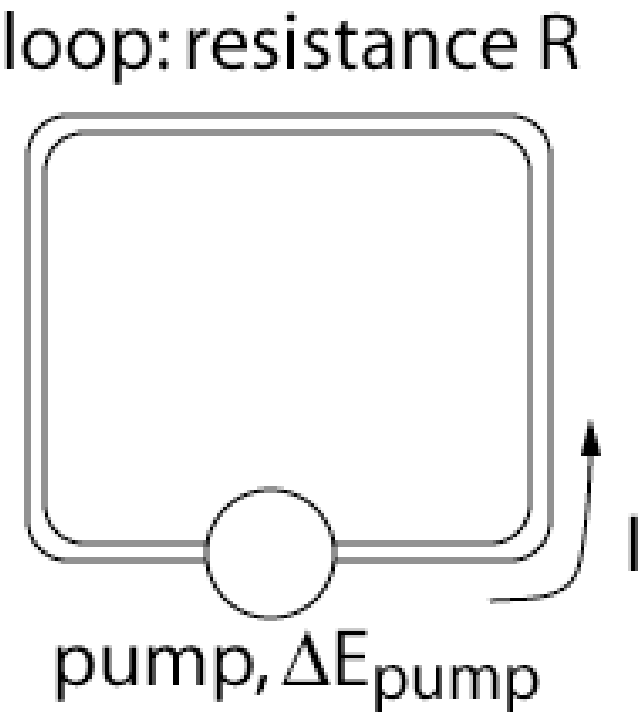
energy/volume x volume/time = power

IR x I = power = $\mathcal{P}$

$$\boxed{\mathcal{P} = I^2 R}$$

Power in Simple Pump

- power: energy/time in from pump = energy lost due to friction in pipe

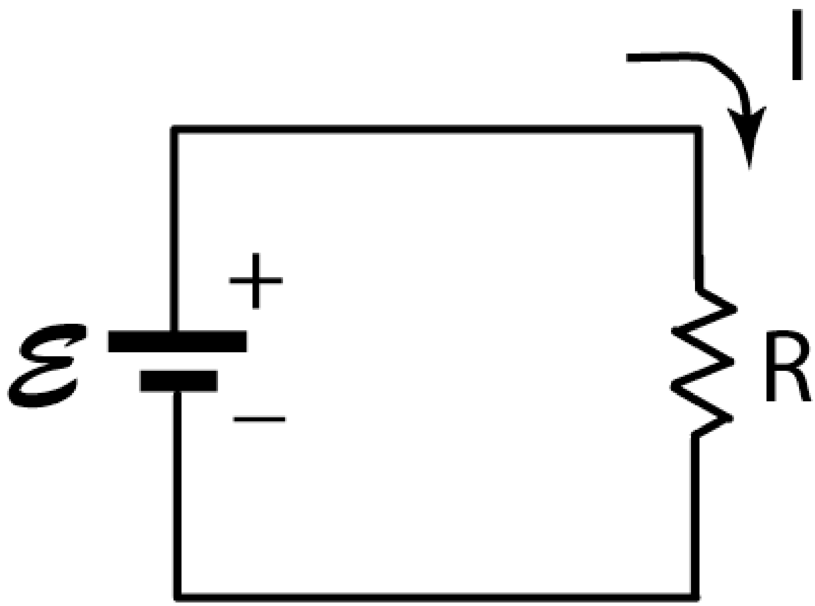


$$\frac{\Delta E_{pump}}{vol} = IR$$

$$\mathcal{P} = I^2 R$$

Simple Resistor Circuit

- pump + circuit is analogous to battery plus resistor!



current: I

emf: $\mathcal{E} = IR$

power: $\mathcal{P} = I^2R$

Resistor Circuit - PRS

Given current/power relations, which of the following is not true?

A. $R = \mathcal{E}/I$

B. $\mathcal{P} = \mathcal{E}^2/R$

C. $\mathcal{P} = \mathcal{E}I$

D. $R = \mathcal{E}I$

E. $I = \mathcal{E}/R$

current: I

emf: $\mathcal{E} = IR$

power: $\mathcal{P} = I^2R$

Electric Forces

- a particle with electric charge q in an electric field strength E feels a force

$$F = qE$$

- the electric force is the gradient of the electric potential energy U:

$$F = \frac{dU}{dx}$$

Electric Forces

- we define the electric potential as the electric potential energy of a charge divided by the charge:

$$V = \frac{U}{q}$$

- electric potential is commonly called voltage

1 Volt = 1 Joule/Coulomb

Electric Forces

- one Coulomb is a very large amount of charge: the charge on an electron is 1.6×10^{-19} Coulomb
- putting it all together we have

$$E = \frac{dV}{dx}$$

- units of electric field, then are Volts per meter (V/m = N/C)

Electric Units

If you just remember the units

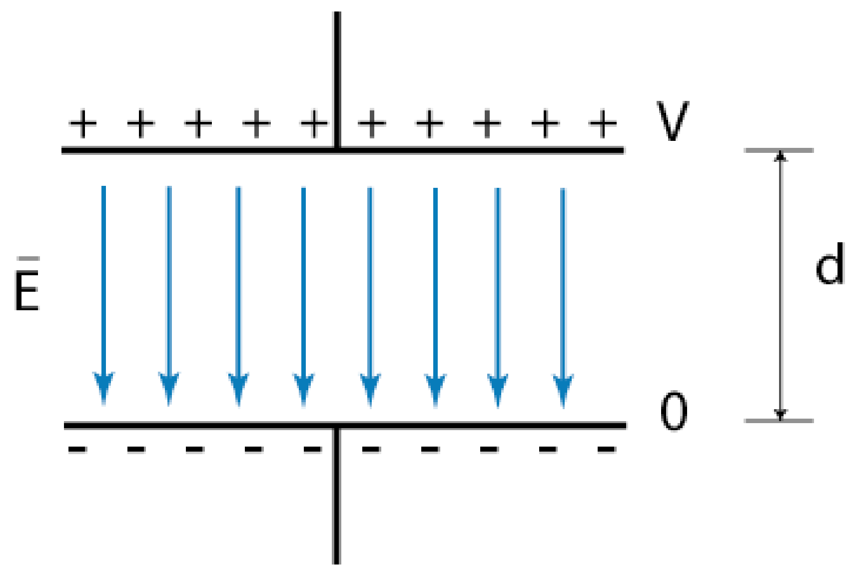
$$1\frac{V}{m} = 1\frac{N}{C}$$

you can get all the equations:

$$F = qE \qquad V = \frac{U}{q} \qquad E = \frac{dV}{dx}$$

Example: Capacitor

- consider two parallel metal plates arranged to have equal and opposite charge: a capacitor

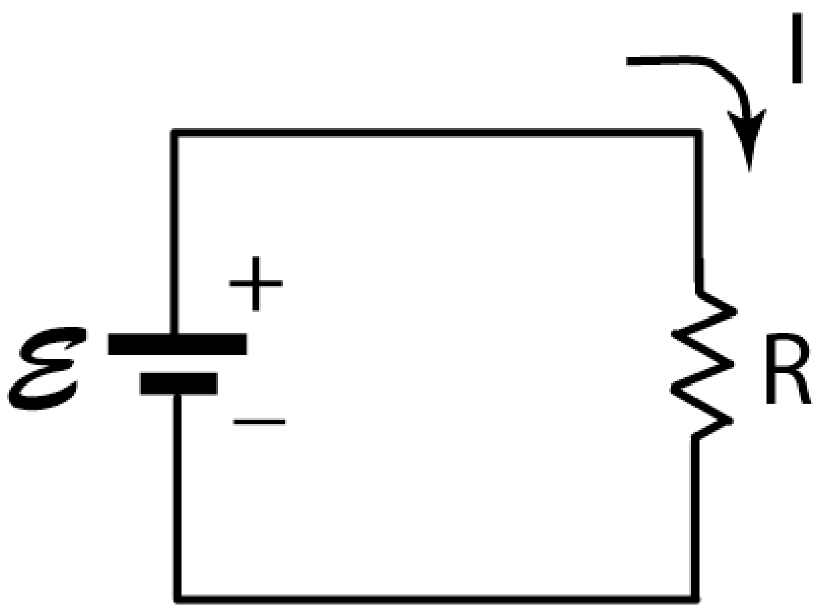


$$F = qE = q\frac{dV}{dx}$$

$$\Rightarrow E = \frac{V}{d}$$

Simple Electric Circuit

- emf: "electro motive force": battery or generator that introduces a constant potential difference in a circuit
- analogous to a pump in a fluid circuit!



current: Coulombs/sec
= Amperes (amps)

resistance: volts/amps
= V-s/C = Ohm (Ω)

Electric Power

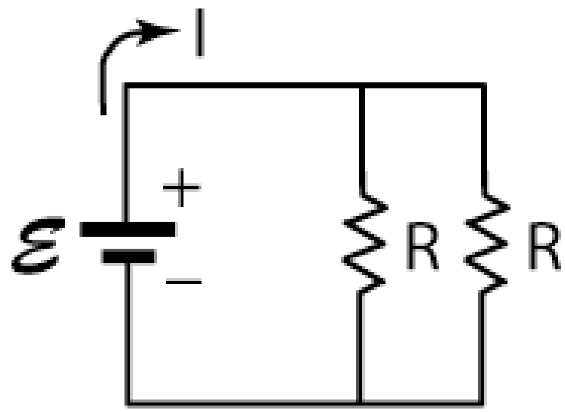
- voltage drop across resistor: energy lost to heat
- power comes from battery/generator emf

$$\mathcal{P} = I\mathcal{E} = I^2R$$

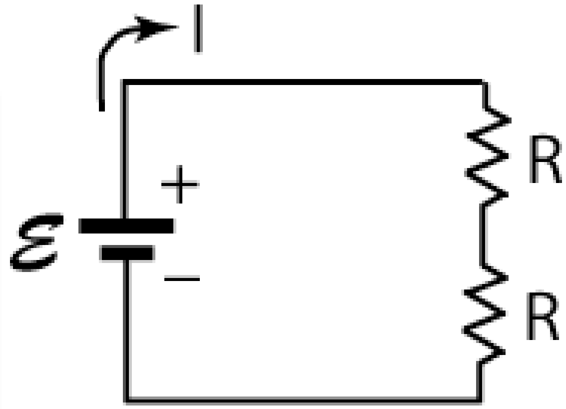
- this is why we transmit power at high voltage!

Parallel/Series

- current in circuit depends on whether resistors are arranged in series or in parallel:



$$I_{para} = \frac{2\mathcal{E}}{R}$$



$$I_{series} = \frac{\mathcal{E}}{2R}$$

Key: think about it the same way as in a fluid circuit!!

Big difference in equivalent resistance!

Parallel/Series

equivalent resistance:

parallel

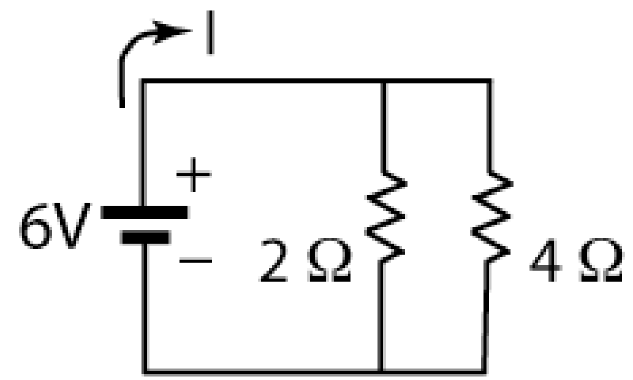
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots}$$

series

$$R_{eq} = R_1 + R_2 + \ldots$$

Series/Parallel - PRS

What is the total resistance of this combination?



A. 6 ohm

B. 2 ohm

C. 4 ohm

D. 1/2 ohm

E. 4/3 ohm

parallel

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots}$$

series

$$R_{eq} = R_1 + R_2 + \ldots$$