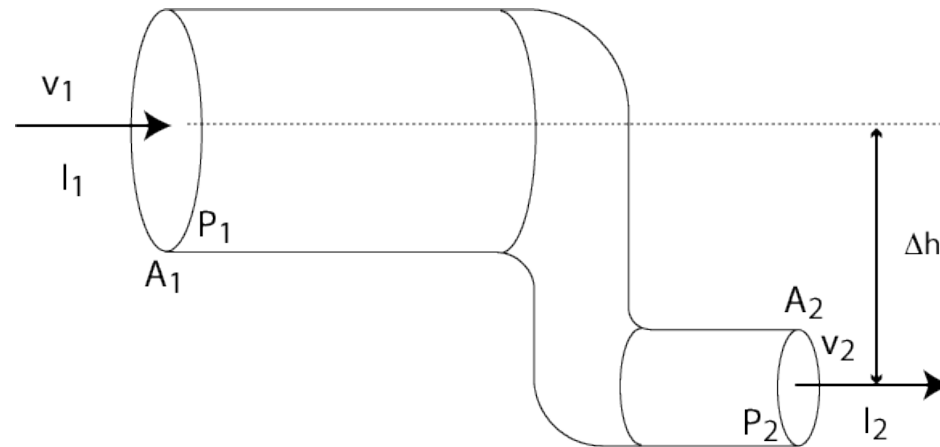


Circuits: Linear Transport

Physics 7B - Lecture 8
Prof. Conway

Last Week's Quiz



1. by the continuity equation $A_2v_2=A_1v_1$, so $v_2=4v_1$; since $I_1=A_1v_1$, we get $v_2 = 4I_1/A_1 = 4(0.2 \text{ m}^3/\text{s})/(0.4 \text{ m}^2) = 2 \text{ m/s}$
2. $(P_2-P_1)+1/2\rho(v_2^2-v_1^2)+\rho g(h_2-h_1)=0$ so we get that $P_2 = P_1 - 1/2\rho(v_2^2-v_1^2) - \rho g(h_2-h_1)$
3. $P_2 = [(200) - 0.5(1000)(2^2-0.5^2) - (1000)(10)(-1)] \text{ Pa}$
 $= (200 - 1875 + 10000) \text{ Pa} = 8325 \text{ Pa}$

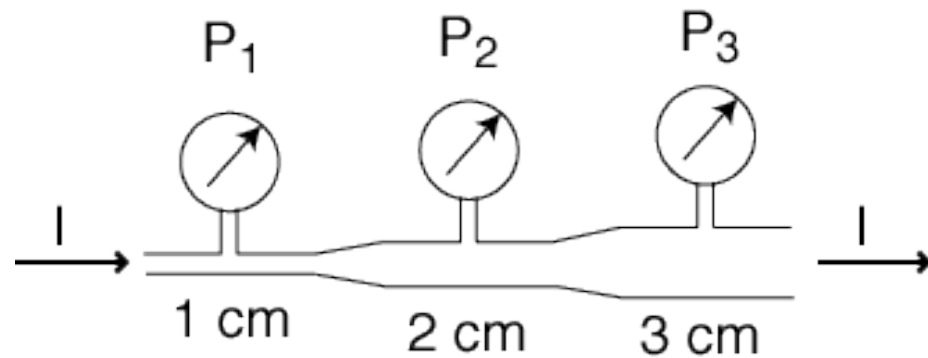
Fluid Flow PRS

Rank the pressures:

A. $P_1 = P_2 = P_3$

B. $P_1 > P_2 > P_3$

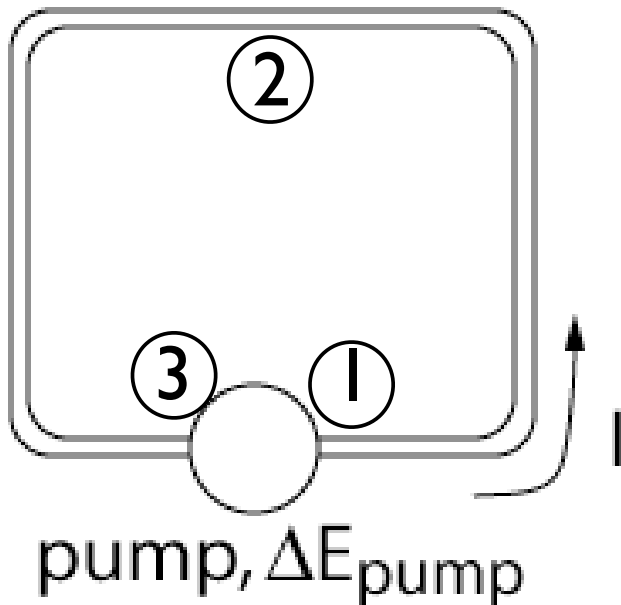
C. $P_1 < P_2 < P_3$



Simple Pump Circuit

- have a pump in a fluid circuit: what is the flow rate given R?

loop: resistance R



$$\Delta P + \cancel{\Delta(\rho gh)} + \cancel{\Delta\left(\frac{1}{2}\rho v^2\right)} = \frac{\Delta E_{pump}}{vol} - IR$$

$$P_2 - P_1 = -IR/2$$

$$P_3 - P_2 = -IR/2$$

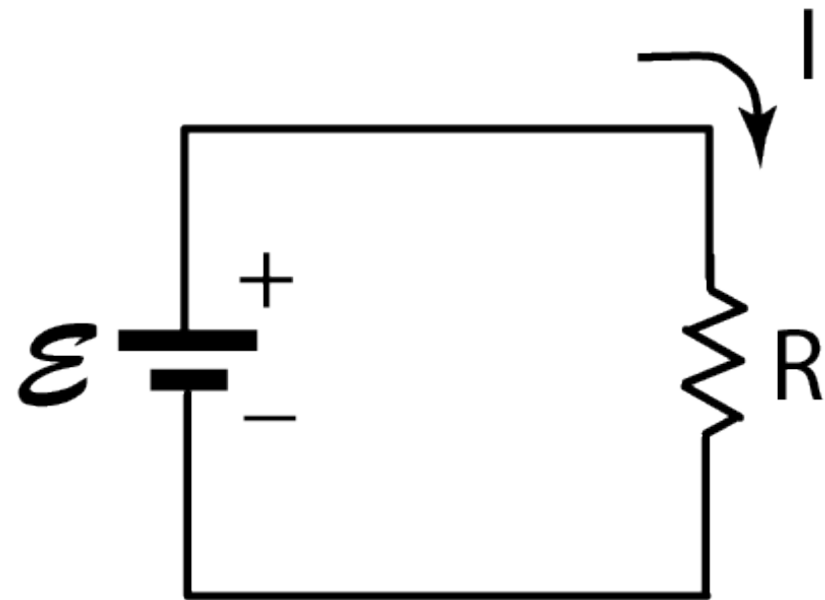
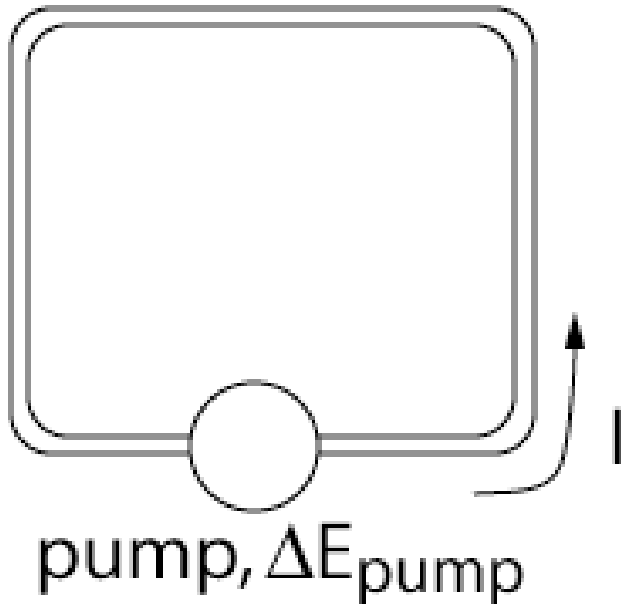
$$P_1 - P_3 = \Delta E_{pump}/vol$$

$$\Delta E_{pump}/vol = IR$$

Simple Resistor Circuit

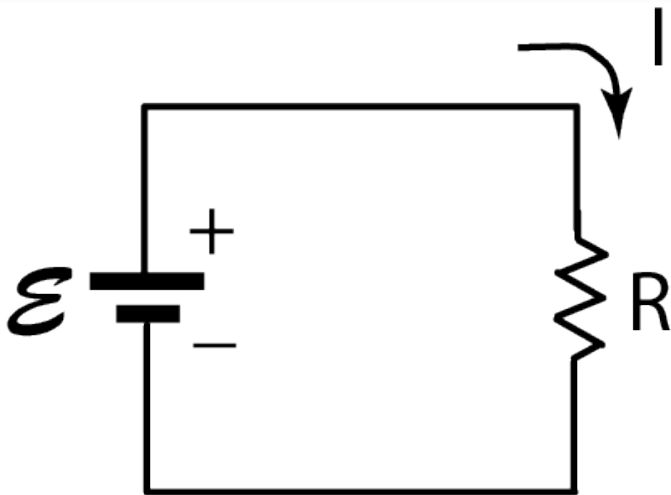
- pump + circuit is analogous to battery plus resistor!

loop: resistance R



Simple Circuit Rules

1. the current in a simple loop is the same everywhere
2. the voltage at all points connected by a wire is the same
3. you can set $V = 0$ any where you want in the circuit
4. if you go through a battery, the voltage changes by E
5. if you go through a resistor, the voltage changes by IR



current: I

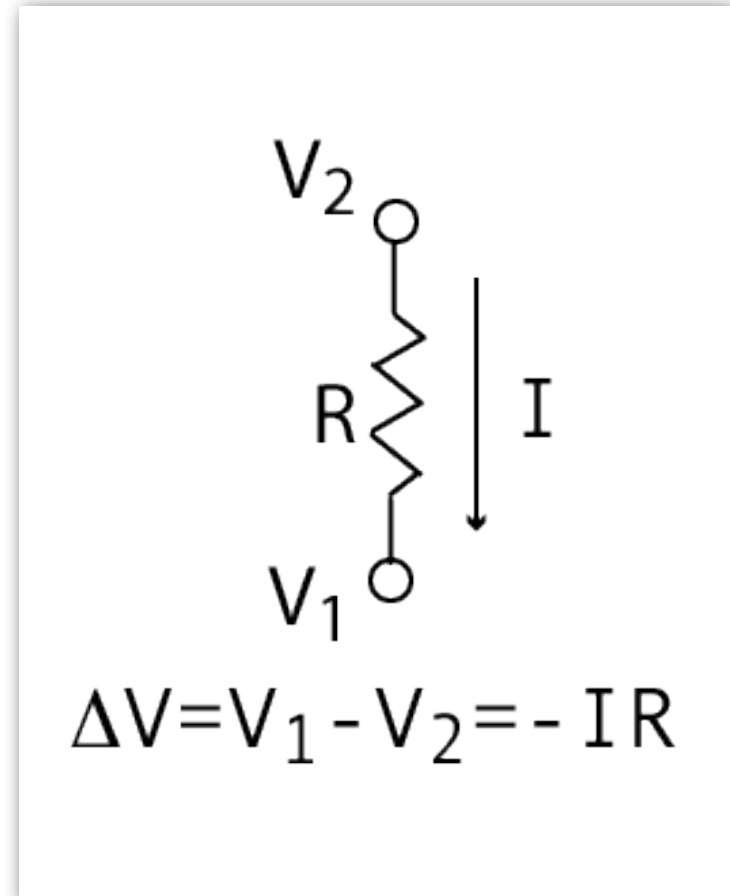
emf: $\mathcal{E} = IR$

power: $P = I^2 R$

Simple Circuit Rules

- If you have a resistor, and you know that there is a voltage drop across it of ΔV , then there must be a current in it of

$$I = \frac{\Delta V}{R}$$

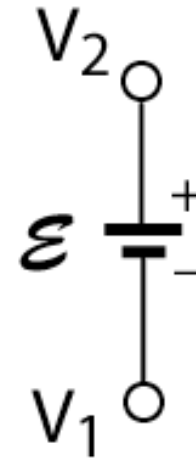


Also, if you know the current you can get ΔV !

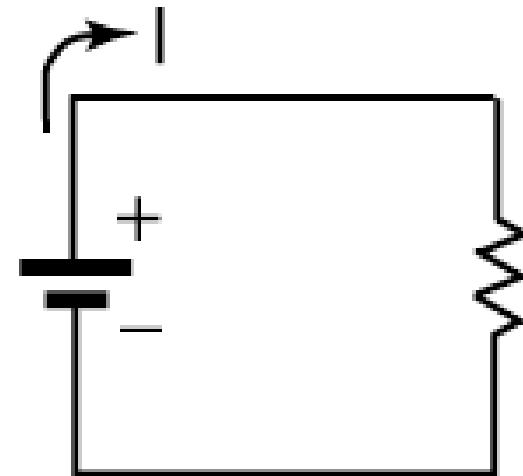
Complete Circuit

- a battery just gives you a change in voltage equal to its emf
- as you trace around a complete circuit, the voltage goes up through a battery and down through a resistor (in the direction of the current)

$$\Delta V = \mathcal{E} - IR = 0 \Rightarrow \mathcal{E} = IR$$



$$\Delta V = V_2 - V_1 = \mathcal{E}$$



Voltage in a Series Circuit

- let's analyze the voltage in a simple circuit:

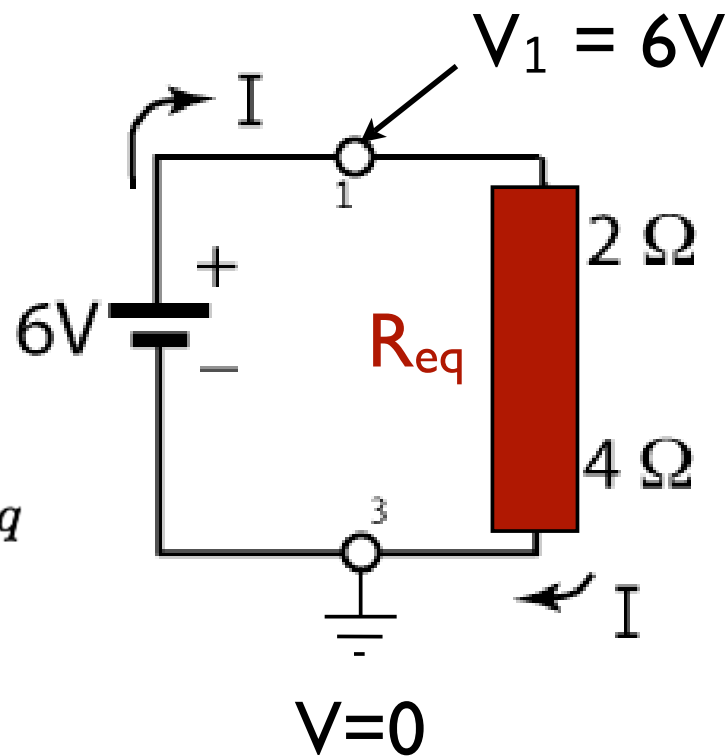
$$\mathcal{E} = IR_{eq}$$

$$V_2 - V_1 = -I(2\Omega)$$

$$V_3 - V_2 = -I(4\Omega)$$

$$\Rightarrow V_1 - V_3 = \mathcal{E} = IR_{eq}$$

$$R_{eq} = 2\Omega + 4\Omega$$



Voltage in a Parallel Circuit

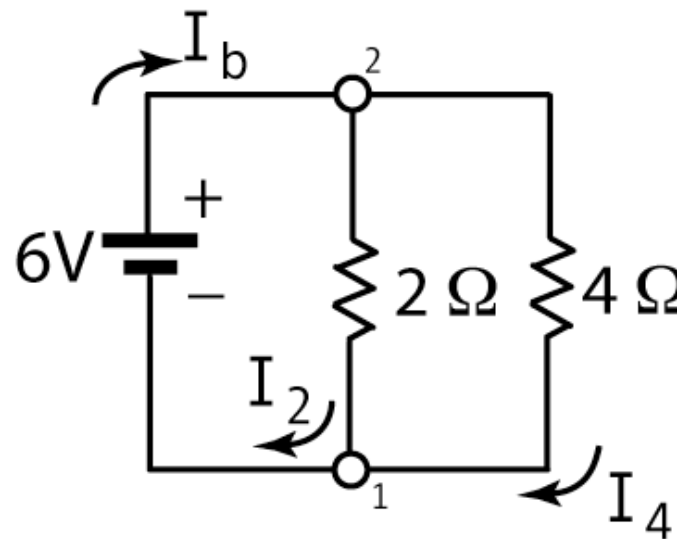
- now let's analyze a parallel circuit: only two voltages to worry about!

$$I_2 + I_4 = I_b$$

$$I_2 = 3A$$

$$I_4 = 1.5A$$

$$\Rightarrow I_b = 4.5A$$



Or we can use parallel equivalent resistance formula:

$$R_{eq} = \frac{1}{\frac{1}{2\Omega} + \frac{1}{4\Omega}} = \frac{4}{3}\Omega$$

Voltage in a Parallel Circuit

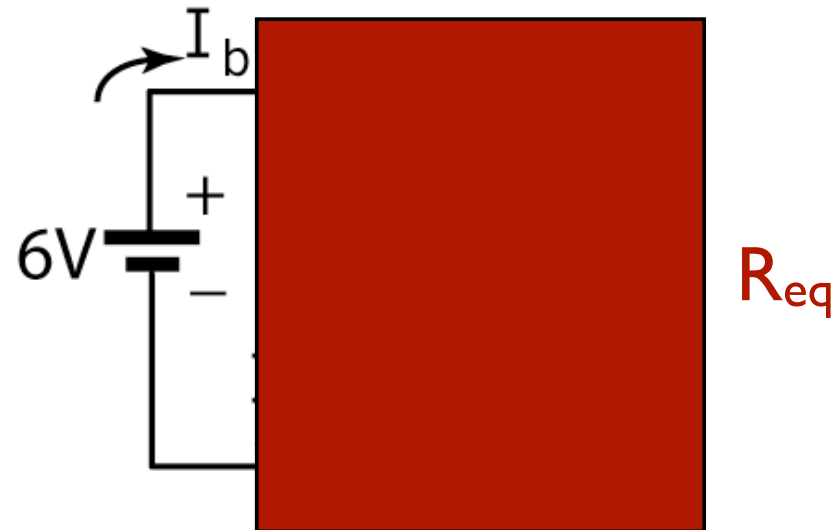
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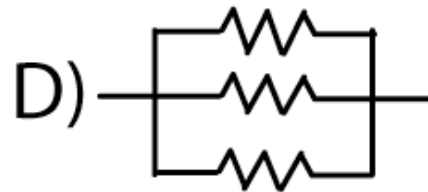
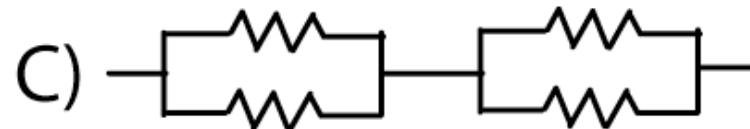
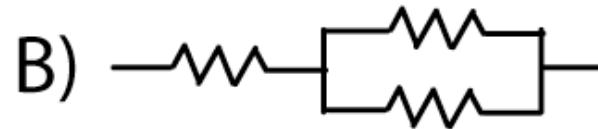
$$\Rightarrow I_b = 4.5A$$



$$I_b = I_2 + I_4 \Rightarrow \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{R_2} + \frac{\mathcal{E}}{R_4} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_2} + \frac{1}{R_4}$$

PRS

- All resistors here have the same resistance. Which set has the least resistance overall?



Quiz

Linear Transport Model

- fluid in a resistive pipe/circuit:

$$\Delta(\text{total head}) = -I_F R_F$$

- electrical current through a resistor:

$$\Delta V = -I_E R_E$$

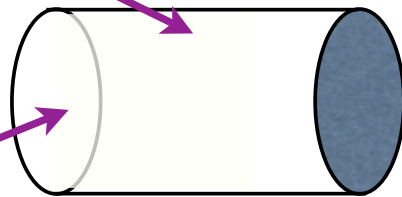
mind the
negative
signs!

- in general we are transporting something (fluid, current, ...) at a certain rate **I** with some resistance **R** to the flow
- we can generalize this concept and address many physical systems quantitatively!

total head includes pressure, height, kinetic energy, pump...

Resistance

- usually the total resistance depends on the material or medium, and the geometry
- intrinsic conductivity (per unit volume): k
- the “fatter” the pipe, the less resistance
- the “longer” the pipe, the more resistance

$$R = \frac{1}{k} \left(\frac{L}{A} \right)$$


The diagram shows a 3D perspective of a yellow cylindrical pipe. A purple arrow labeled 'length' points to the top surface of the pipe. Another purple arrow labeled 'area' points to the left circular face of the pipe.

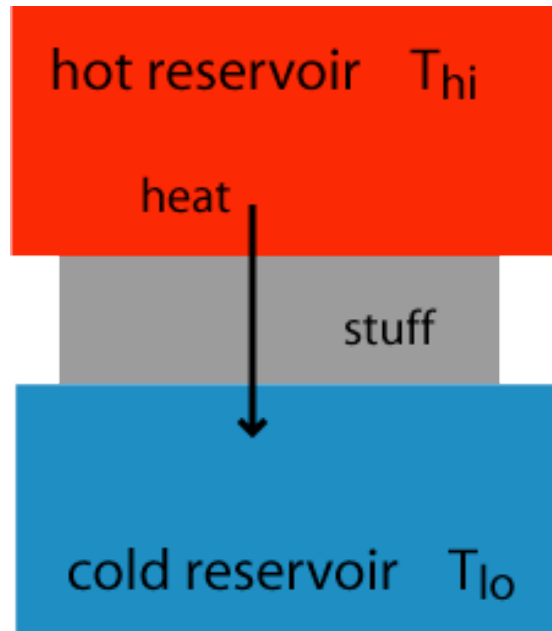
conductivity

length

area

Example: Heat Flow

- simple heat reservoir system: how much heat energy flows per unit time?



energy transport equation:

$$\text{heat/time} = \mathcal{P} = \Delta T \times 1/R$$

analogous to electric circuit
with $I = \mathcal{E} \times 1/R$

Thermal Resistivity

- consider thermal insulator of length L , area A
- more heat goes through bigger area
- less heat goes through longer block

$$R_{th} = \rho_{th} \frac{L}{A}$$

$$\rho_{th} = \frac{1}{k_{th}}$$

(resistivity is the inverse
of conductivity)

diamond is the best
thermal conductor
and the best
electrical resistor

Thermal Resistivity - PRS

Given the equations at the right for heat transfer, the units of k_{th} are

A. W/m^2

B. $W/^\circ K$

C. $W/(^\circ K-m)$

D. $^\circ K/W$

E. $(^\circ K-m/W)$

$$R_{th} = \frac{1}{k_{th}} \left(\frac{L}{A} \right)$$

$$\mathcal{P} = \frac{dQ}{dt} = \Delta T \left(\frac{1}{R_{th}} \right)$$

Home Heating – PRS

In the winter time,

- A. the warmer you keep your house the less energy you consume in the long run
- B. it's more energy-efficient to leave your home thermostat at one comfortable temperature all day
- C. if you let your house get cold while you are not there in the day time, or at night, it takes more energy to heat it back up
- D. the lower the temperature you keep your house, the less money you'll spend on energy for heating it
- E. a certain amount of heat energy always escapes to the outside, no matter what your thermostat is set to

Example: Diffusion

- Imagine a situation where there is a high concentration C of some particle on one side of a permeable membrane or barrier of thickness t , and a low concentration on the other
- What is the rate of particles crossing the membrane?
- particle current $I = (\text{number of particles})/(\text{second})$

$$\Delta C = -IR_c$$
$$R_c = \frac{1}{D} \left(\frac{t}{A} \right)$$

D: diffusion constant of material

Summary

quantity	fluid	electrical	heat
transported	volume	charge	heat
current	I (m^3/s)	I (C/s)	\mathcal{P} (W)
potential	(head)	voltage V	ΔT
resistance	R (J-s/ m^6)	R (Ω)	R (K/W)
power	$I^2 R$	$I^2 R$	-