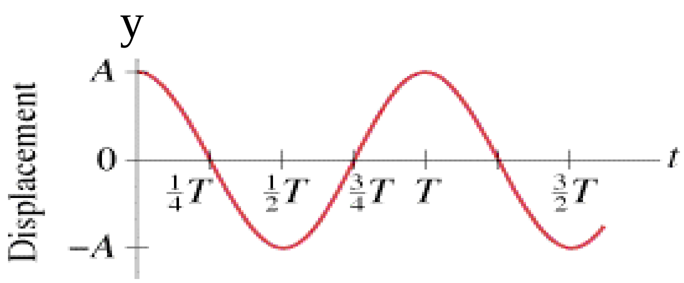
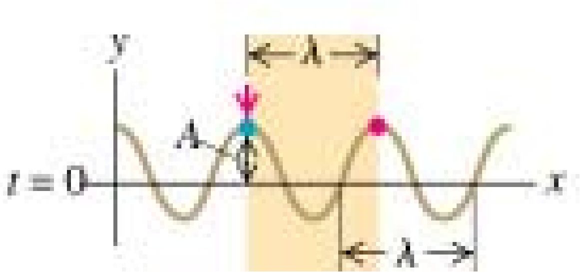
# Graphical Representation




*y vs t:*
*Motion of a single point in the medium*



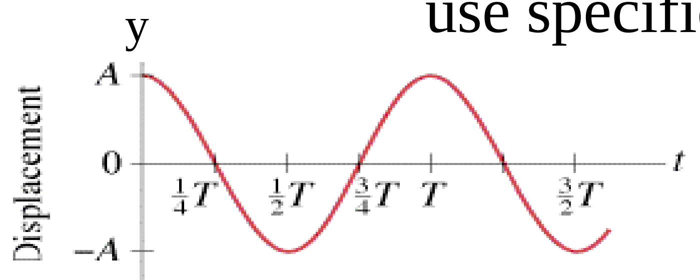

*y vs x:*
*Snap shot of the whole wave at one instant*

# Phase

General expression: $y = A\cos(kx - \omega t + \phi)$

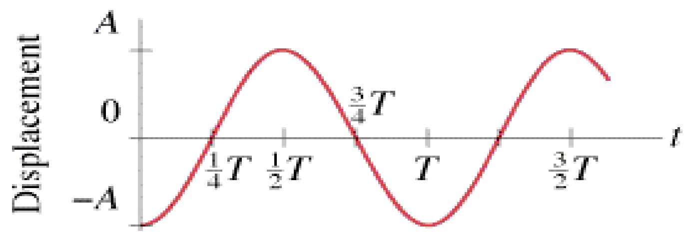
use specific points to determine $\phi$

*x=0*



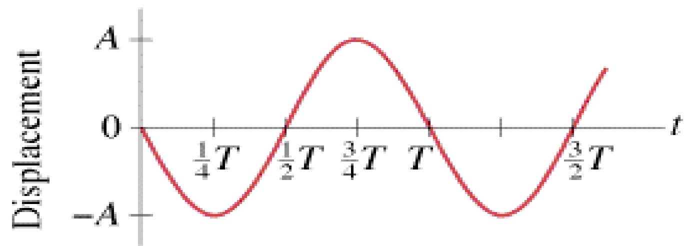

$y = A\cos(-\omega t)$
$= A\cos\omega t$

*x=λ/2*



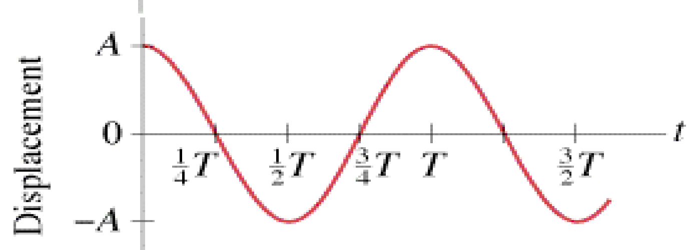

$y = A\cos(\pi - \omega t)$

*x=3λ/4*




$y = A\cos(3\pi/2 - \omega t)$

*x=λ*




$y = A\cos(2\pi - \omega t)$
$= A\cos\omega t$

# Particle Velocity and Acceleration

Recall wave velocity: $v=\lambda f=\omega/k$

$$y(x,t) = A\cos(kx - \omega t)$$

Transverse velocity of a particle

$$v_y = \frac{\partial y(x,t)}{\partial t} = \omega A \sin(kx - \omega t)$$

Generally true

Acceleration of a particle

$$a_y(x,t) = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\cos(kx - \omega t) = -\omega^2 y(x,t)$$

Proportional to displacement

Opposite direction

# Wave Equation

Since

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 y(x,t)$$

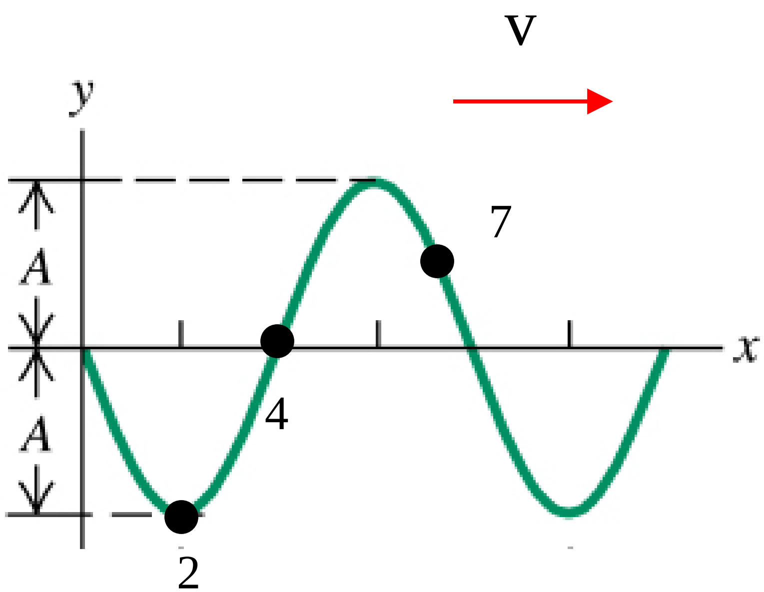
$$\frac{\partial^2 y(x,t)}{\partial x^2} = -k^2 y(x,t)$$

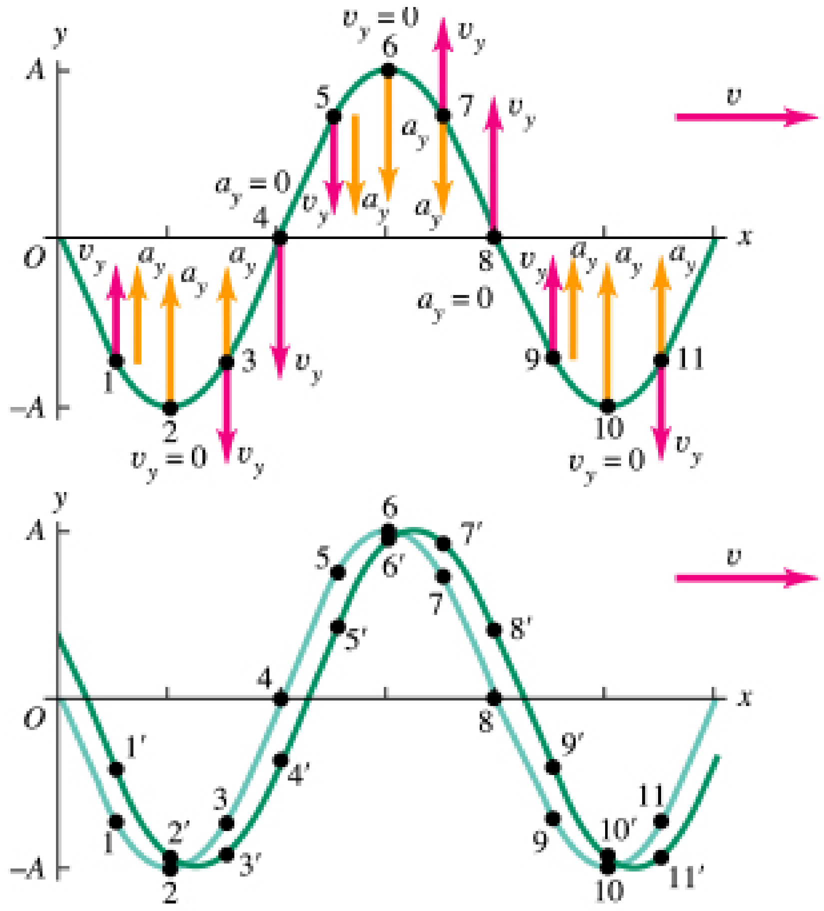
$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

$$= \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

Valid for waves on a string that have any shape.

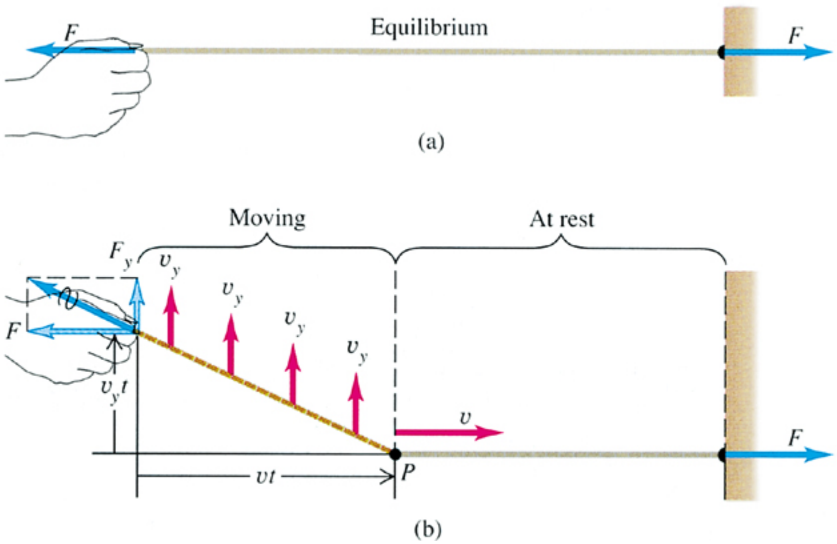
# Wave Snapshots



# 15-4. Speed of a Transverse Wave

**Method 1: Impulse-momentum theorem**




Please read text on your own.

$$v = \sqrt{\frac{F}{\mu}}, \text{ speed of a transverse wave on a string}$$

$F$: tension in string, N

$\mu = m/L$: linear mass density

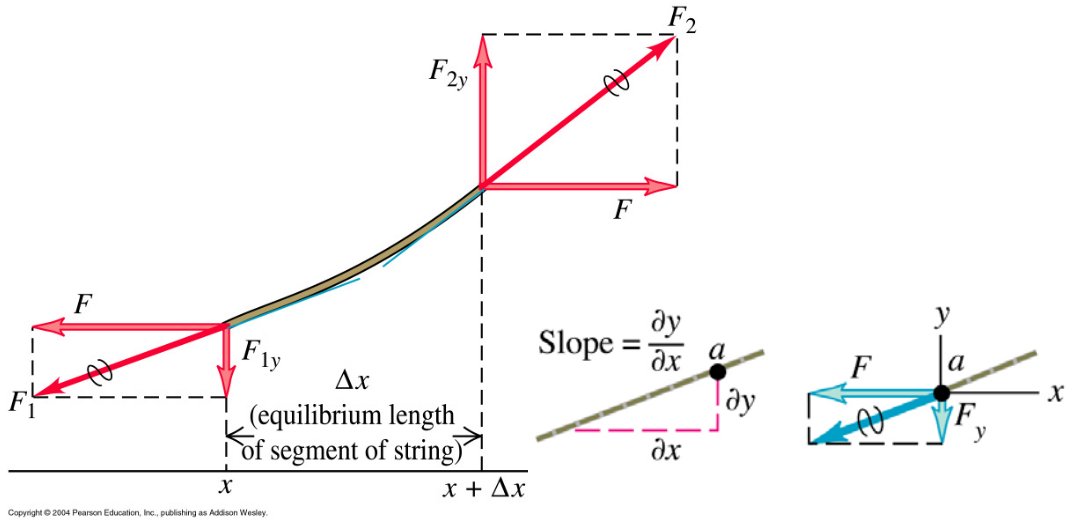
# Speed of a Transverse Wave

**Method 2: Newton's second law**

$$F_y = F_{1y} + F_{2y} = F\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right]$$

$$F_y = ma_y = \mu \Delta x \frac{\partial^2 y}{\partial t^2}$$

$$\lim_{\Delta x \to 0} \frac{\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right]}{\Delta x} = \frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}$$




Wave equation: $$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$v = \sqrt{\frac{F}{\mu}}$$

$F$: tension in string, N

$\mu = m/L$: linear mass density